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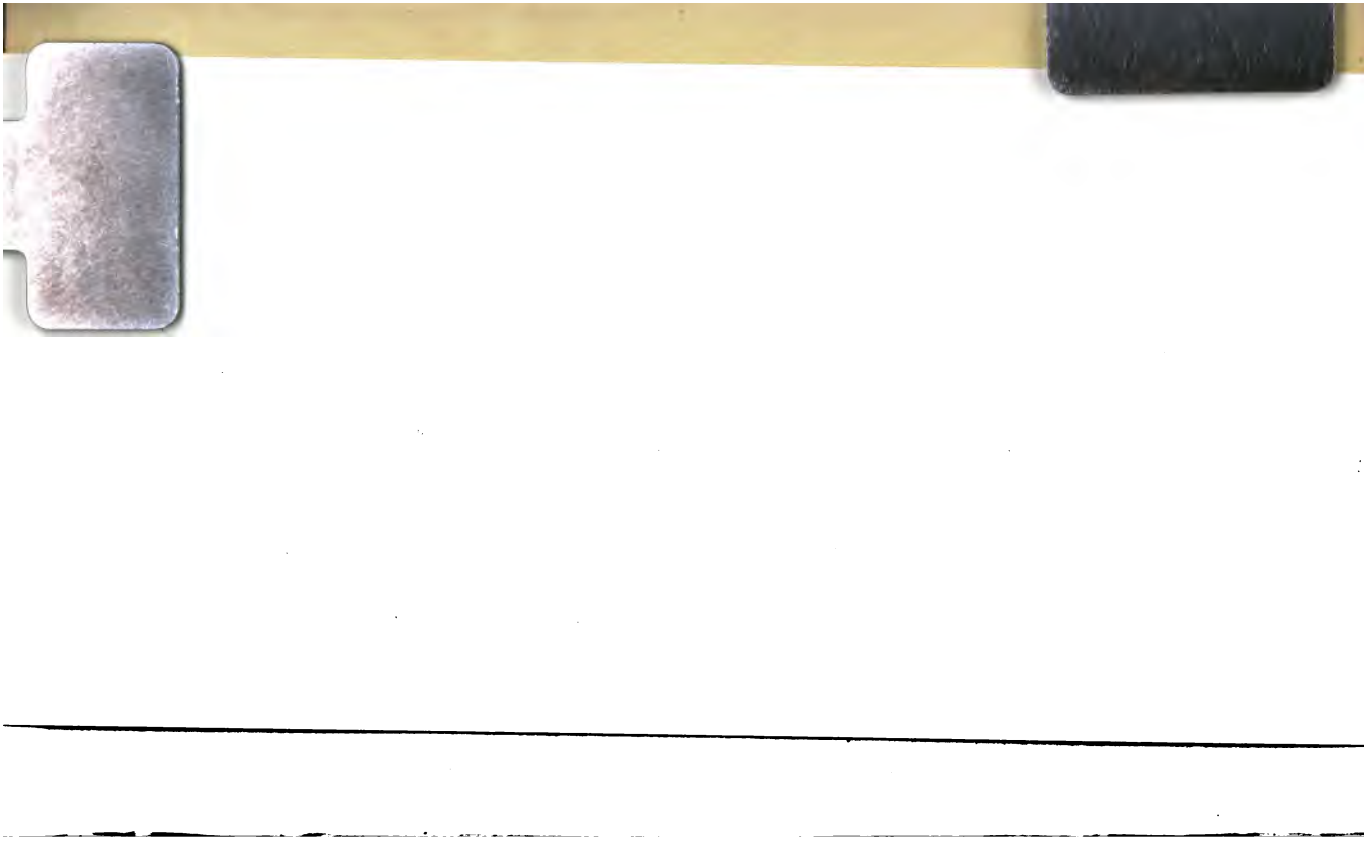

April 21
1883.

G. P. Bond Esq

with compliments of

The Author

METHODS OF LOCATION

OR MODES OF

DESCRIBING AND ADJUSTING

RAILWAY CURVES AND TANGENTS,

AS PRACTICED BY THE

ENGINEERS OF PENNSYLVANIA,

BY SAMUEL W. MIFFLIN,

CIVIL ENGINEER.

THIRD EDITION REVISED AND GREATLY IMPROVED.

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PREFACE

TO THE FIRST EDITION.

In submitting this work to the public, I do not wish to claim for myself its exclusive authorship. A few of the first solutions were the work of my comrades on the Pennsylvania Railway; these falling into my hands suggested the idea of a complete series of Geometric solutions applicable to all cases that might occur.

In such a work I considered it of importance to dispense with all difficult calculations, and even with tabular statements, which cannot be committed to memory. In this I have fortunately succeeded; there is nothing in the following pages which may not be remembered by an assistant after a short practice, and executed in the field *even if the book be left at home.*

There are a few instances, however, in which a table of chords may facilitate operations, and one has therefore been placed upon the last page for the use of those who prefer it.

EXPLANATIONS.

1. Since all the curves described in this work are circular, the words curve and circle will be used indiscriminately.
2. All measurements are referred to some chord of convenient length as a unit, which may be either the common four pole chain of 100 links, or one of 100 feet, and for brevity sake, the word chain will be used to designate such chord.
3. The angle subtended by the above chord at the centre of the circle is called the degree of curvature, or simply the curvature. A. F. B. Fig. 1.
4. The letters *m* and *n* are used to express degrees of curvature, and when both are used, *m* is the greatest, that is, it belongs to the smallest circle.
5. A central angle is that which a chord subtends at the centre of the circle. A. F. B. Fig 1.
6. A circumferential angle is that which a chord subtends at any point in the circumference. A. E. B. Fig. 1.
7. A tangential angle is the smallest angle made by a chord at its extremity, with a tangent to the curve at that extremity. G. A. B. Fig. 1.
8. A compound curve is composed of two curves of different radii turning in the same direction, having a common tangent at their point of meeting.
9. This point of meeting is called the point of com pound curvature, or simply P. C. C.
10. A reversed curve is composed of two curves turning in opposite directions and having a common tangent at their point of meeting.

11. This point is called P. R. C., or point of reversed curvature.
12. A differential curve is one whose radius is equal to the difference between the radii of any two curves to which it is applied.
13. An integral curve is one whose radius equals the sum of the radii of two other curves.
14. Equivalent arcs or curves are such as subtend equal central angles.
15. Corresponding points in different circles are any points, where the tangents and of course the radii are parallel.
16. The terms origin and termination are used in reference to the course of location. The termination of a tangent being the point where a curve is commenced, and the origin of the next tangent the point where the curve terminates.
17. The origin is also called the point of curve, or point of tangent, or simply P. C. or P. T.

April 21
1863

G. P. Bond Esq

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8. If from a point without a circle there be drawn two lines one of which touches the circle and the other cuts it, the rectangle of the whole secant line and the part without the circle is equal to the square of the tangent.

The following Propositions, although not strictly correct, are sufficiently so for all purposes of Location.

9. The central, circumferential, and tangential angles of chords of unequal lengths are directly as the lengths.

10. The radii of circles are directly as their degrees of curvature.

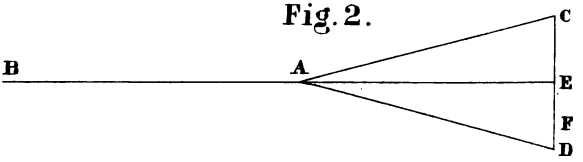
11. The radius of a circle is half the circumference divided by 3.1416.

12. If the chord of one degree be taken as a unit, the circumference may be considered equal to 360.

Hence, the radius is equal to $\frac{180}{3.1416} = 57.30$ and by proposition 10 the radius of any other circle is $\frac{57.3}{m}$

NOTE.—The 9th, 10th and 12th propositions are true to the second place of decimals, so long as m is not greater than 10°, which is double what is required in ordinary cases.

Fig. 2.



PRELIMINARY EXERCISES.

In the Use of the Transit.

THE Transit is an instrument invented and manufactured by W. J. Young of Philadelphia. It is in many respects more convenient than the Goniometer or Theodolite, and being the instrument to which I have been most accustomed, I have adapted the phraseology of this treatise to its use. There is, however, no difficulty in solving all the propositions in this series with either of the other instruments above mentioned.

PROPOSITION I.

To adjust the vertical hair of the Telescope so that the Lines of sight forward and backwards shall be parts of the same Straight Line.

CHOOSE a piece of perfectly level ground, from 500 to 800 feet long, and clear of all obstruction to the sight, set the instrument in the middle as at A, Fig. 2d, level and clamp it, and with the tangent screws bring the sight to bear upon a chain-pin or any other suitable object which an assistant must hold at B.

Then reverse the *Telescope* on its axis and set up another pin in the opposite direction, and at the same distance as B is from A: if the instrument be out of adjustment this will not fall in the line AB produced but on one side of it as at C.

Now unscrew the clamp and without touching the Telescope reverse the *Transit* on its axis, and fix the sight upon B as before, and screw up the clamp.—Again reverse the *Telescope* and set up a third pin, which will now fall upon the point D precisely, as far to the right of AE, as C is to the left. Divide accurately the distance between C and D, and set up a fourth pin at E; B, A, and E will then be in the same straight line. Now remove the pin from C, and set it up at F, precisely in the middle of E D, and with the adjusting pin remove the vertical hair until it coincides

with F, then with the tangent screws fix the sight upon E and reverse the *Telescope.* If the operation has been carefully performed, the sight will strike the chain-pin at B and the adjustment is effected.

It generally happens, however, that a second slight movement of the hair is necessary to perfect the operation, which should be tested by several reversions on the axis of both Transit and Telescope, until the coincidence of the hair with B and E is fully established.

PROPOSITION II.

To discover whether the Telescope revolves truly in the Meridian.

After completing the adjustment by the last proposition, choose a steeple or any other lofty object upon whose top a steady and accurate sight can be obtained; set the instrument as near to its base as possible, and after leveling and clamping it, fix the sight upon the top of the object and turn the head of the Telescope in the opposite direction, so as to bear upon the ground at some convenient distance from the instrument, and set up a pin.

Then reverse the *Transit* on its axis and take sight as before to the top of the steeple: again turn the head of the Telescope towards the pin just set up, and if the vertical hair coincides with it, the instrument is sound, but if not, half the distance between them will be the error. When the Transit is furnished with two leveling tubes at right angles to each other this defect may be remedied by dividing the error as in prop. 1., Fig. 2, and adjusting the cross tube to the point thus found and the top of the steeple, but as this operation is apt to throw the upright hair out of a vertical position so that only the intersection of the two hairs can be used, it is better if practicable to send the instrument to the maker for repairs.

Fig. 3.

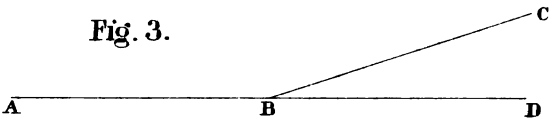


Fig. 4.

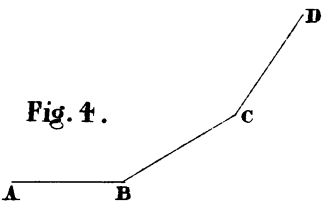
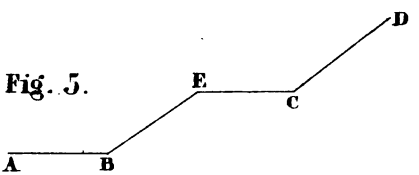


Fig. 5.



PROPOSITION III.

To Measure the Angle between any two Lines or Deflection from one Line to another.

In performing this operation, it is necessary to bear in mind the course of survey: thus, if the survey proceeds from A to B, Fig. 3d, and afterwards from B to C, then the angle required will be CBD, but if the course of survey is from D to B, and afterwards from B to C, then the angle required is ABC. To measure the angle proceed as follows: place the instrument over B, and set the index to Zero, then take a back sight from B to A, and *reverse* the *Telescope:* turn the rack until the sight bears upon C, and the index will then show the angle DBC required.

Case 2d.—*When the intersection of two lines* AB *and* CD, *Fig.* 4th, *are inaccessible.* Place the instrument over B, set the index to Zero, and take a back sight to A, then *reverse the Telescope*, turn the rack and take sight upon C; screw the rack fast, and remove the instrument to C; take a back sight from C to B: again *reverse the Telescope* and turn the rack till the sight falls upon D. The index will then show the required angle.

Case 3d.—*When the ground is so encumbered that no part of* CD *can be seen from* AB. Take a point E, Fig. 5th, from which both lines may be seen, and deflect from AB to BE in the same manner, as from AB to BC in the last case, then remove to E, and continue the process by deflecting from BE to EC, and so on, until the instrument is brought into the position CD, the index will then show the angle as before.

Fig. 6.

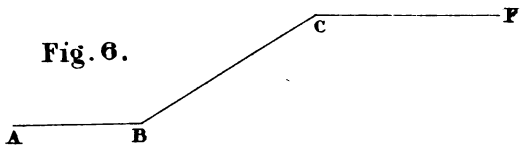
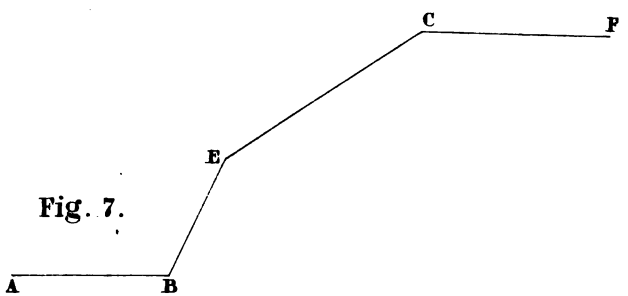


Fig. 7.

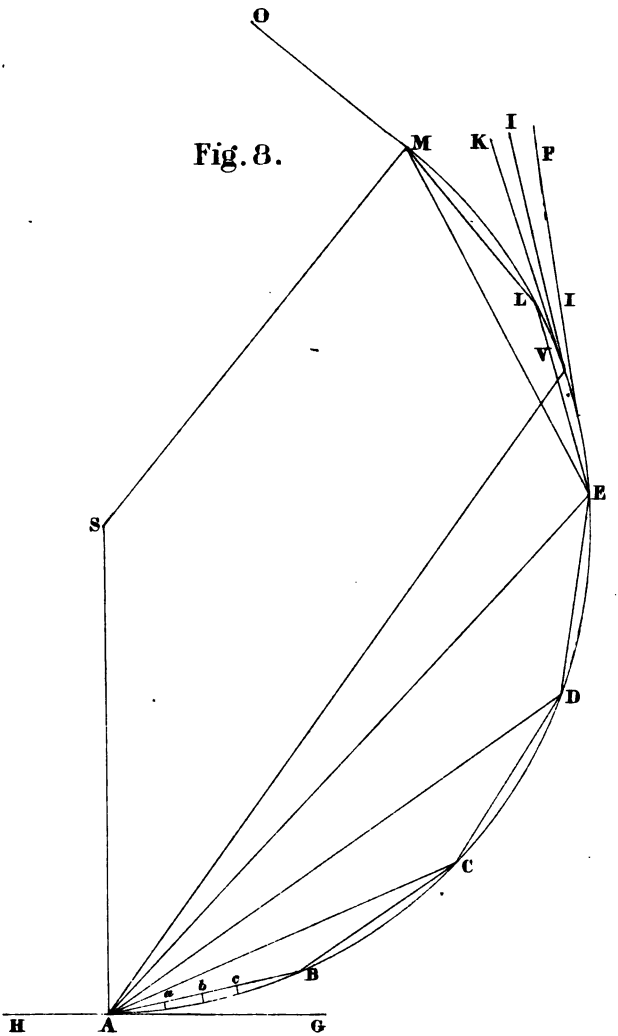


PROPOSITION IV.

To run a Line from a given Point, parallel to a given line.

LET C, Fig. 6th and 7th, be the point, and AB the given line, set the instrument at B and proceed as directed in the second and third cases of the last proposition until arriving at C, then turn the index back to Zero and the glass will then be in the line CF parallel to AB.

Fig. 8.



PROPOSITION V.

To describe a circle on the ground with the Transit and Chain from any point in a given Tangent with any degree of curvature or central angle.

Let HG, Fig. 8, be the tangent, and A the point. Place the instrument over A, set the index to Zero, and take a back sight to H. Then reverse the Telescope and turn the index till it shows the *tangential*, i. e. *half the central* angle, measure AB with the chain, and fix B with the telescope.

Make BAC, CAD, and DAE, successively, equal to the *tangential* or *circumferential* angle and measure BC, CD, and DE as before; B, C, D and E will be points in the curve.

When E is found, the index shows the whole angle EAG; then remove the instrument to E, take a back sight to A, and turn the index till it shows twice the angle EAG; then turn the glass towards F, and EF will be the course of a tangent from E. This is called completing the tangent.

If it be necessary to continue the curve, repeat the above process by adding the tangential angle successively, and measuring EL, LM, &c. as before. To complete the tangent at M, after taking a back sight from M to E, it is necessary to add only the tangential angle FEM to what is already shown by the index, which will then show the whole central angle ASM, or exterior angle of the tangents HA and MO. But if the proper course of the tangents require it to be

drawn from a point between E and L, ascertain by Proposition III, page 11, the angle it would make with EF, and make VEF equal to half that angle, and take EV : EL : : VEF : LEF, then make IVK equal to VEF and VK is the course of the tangent required.

If the angle VEF were known, that is, if the whole exterior angle of VK with HA were known, the point V may be fixed from A, by making VAE equal to VEF, i. e. VAG equal to half the exterior angle of VK with AG, and measuring EV as before.

Intermediate points *a*, *b*, *c*, must be found by calculating the ordinates by Proposition XVIII, page 45, or they may be found by making the angles proportionate to the distances measured on AB, and setting off *a. b. c.* &c. at right angles, but the ordinates are best suited to an unpractised hand.

The demonstration of the above method is evident from the preliminary proposition.

NOTE. Since the chain EL whose central angle is m contains 100 links, each link of EV will have a central angle of $\frac{m}{100}$

The index should, therefore, be divided into hundredths instead of minutes. More perplexity will be avoided by this simple contrivance than can well be imagined by any one who has not made the experiment.

In all instruments made for the author, Mr. Young has adopted the decimal notation and no one who has used one of these instruments has ever wished to return to the old graduation of degrees and minutes.

To render plainer the working of the last proposition and also to show the best method of keeping a transit book, two examples are inserted below.

In Example 1st., the Station column refers to the points A. B. C. &c., Fig. 8, which in an actual location would be represented by Nos. 1, 2, 3. &c.

The second column shows the distance (50 ft.) of the point A from the last station H. The transit is supposed to be placed at A. the telescope pointing in the direction of the tangent AG. The index as shown in the fourth column stands at 0 which also represents the position of AG. The sixth column contains the magnetic course of the tangent and the column of Remarks informs us that A is the origin of a curve of 4° per hundred feet turning to the left.

A deflection of 2° to the left brings the telescope into the direction AB. and the index stands at 2° while AB. is measured off and the point B fixed. An additional deflection of 2° brings the telescope into the line AC and the index stands at 4° while the point C is adjusted. A third deflection brings the index to 6° for the point D, and to 8° for E.

The transit must now be moved to E and while the index stands at 8° take a back sight to A and then reverse the telescope.

To get the position of the tangent EF. take the difference between the index column at E and the tangent column at A thus 8—0=8 and add this to the index at E, 8+8=16. Turn the index to 16° and the telescope will stand in the position EF.

Continue the curve by adding the deflection (2°) to the tangent (16°) which gives 18° for the index while fixing the point C and 20° for M.

Remove the instrument to M. see that the index keeps its place at 20° while taking a back sight to E. and then reverse the telescope.

From the index at M take the tangent at E and add the remainder to the index for the tangent at M. thus

20—16=4. and 20+4=24°, to which turn the index for the tangent MO.

Example 2nd, is a transcript of Example 1st, as far as station E, but here it is found that the true course of location lies in the direction V K. which by measurement is found to make an angle of 2°. 6 with EF. Divide this angle by the degree of curvature and the result $\frac{2.6}{4}$ =. 65 is the length of the chord EV. when AB, or DE, is 1.00. The deflection for this distance will be 1.3 and the index 17.3 for the point V.

On removing to V. we have 17. 3+17. 3—16= 18° 6 for the tangent VK.—An additional check on the correctness of this operation is found in the column of magnetic courses which should always give *very nearly* the same angular differences as the column of tangents.

EXAMPLE 1st.

stat.	dist.	defn.	index	tangt.	mag. course	remarks
A	50		0	0	N 2 E	P. C. 4° Left
B	100	2°	2°			Dry Meadows
C	100	,,	4			Woodland
D	100	,,	6			do
E	100	,,	8	16	N 14 W	Swamp
L	100	,,	18			Cult. field
M	100	,,	20	24	N 18 W	P. T.
O	100	,,			,,	

EXAMPLE 2nd.

stat.	dist.	defn.	index	tangt.	ma. course	remarks.
A	50		0	0	N 2 E	P. C. 4° Left
B	100	2°	2°			
C	100	,,	4			
D	100	,,	6			
E	100	,,	8	16	N 14 W	
V	65	1. 3	17. 3	18. 6	N 16½ W	P. T.

Fig. 9.

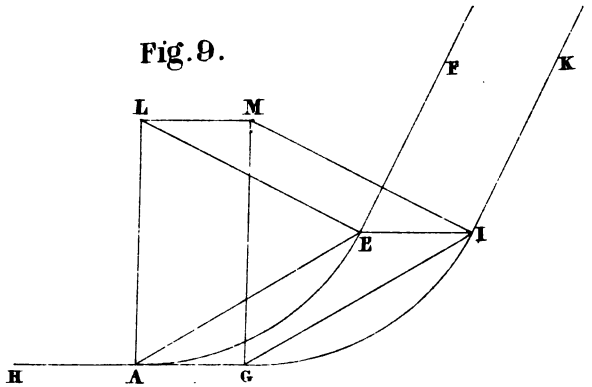
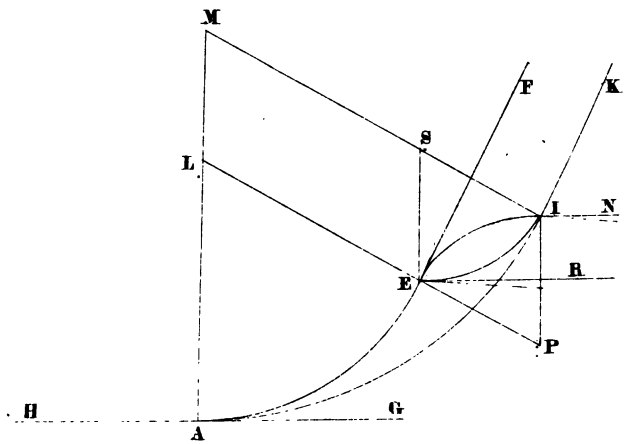


Fig. 10.



PROPOSITION VI.

To change the origin of a Curve so that it shall terminate in a tangent, parallel to a given Tangent.

SUPPOSE the curve AE, Fig. 9th, terminating in the tangent EF to have been described as directed in the preceding Proposition, and that the nature of the ground requires that it should terminate in IK parallel to EF.

Measure the distance EI on a line parallel to AG, and make AG equal to EI, G will be the origin of a curve, similar to AE, that will terminate in IK at the point I.

The parallelism of all the lines drawn from A and E, with similar lines from G and I, sufficiently show the correctness of this method without a formal demonstration.

PROPOSITION VII.

Having a curve AE, *Fig.* 10, *terminating in a tangent* EF, *it is required to find where a curve of a different radius originating in the same point would terminate in a tangent parallel to* EF.

CONSTRUCTION.

LET L be the centre of AE, and on AL produced, take A M equal to the radius of the other circle, draw ES parallel and equal to LM, and from S with the

radius SE describe the differential circle EI, join MS and produce it till it cuts EI in I, then will I be the terminating point required.

For ES being equal and parallel to LM, MS must be equal and parallel to LE; hence, MI is equal to MA, and the angles LMS and ESI are equal to ALE, and therefore, the tangent EF is parallel to IK.

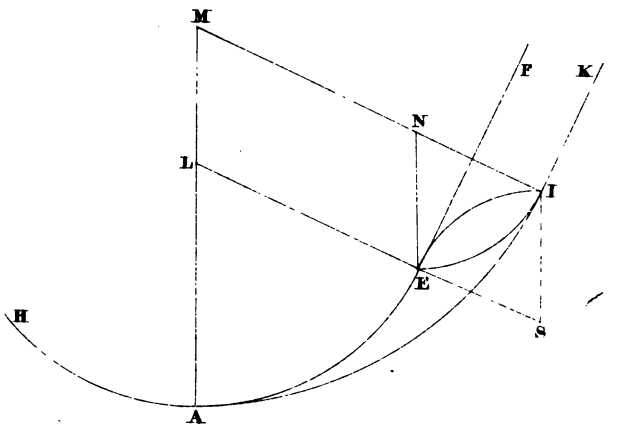
Hence, the field operation is evident. The instrument must be set over E, and the Telescope brought into the position ER parallel to AG. From ER as a tangent describe the differential curve EI, and make it equivalent to ALE, and I will be the point sought.

By completing the parallelogram SEPI, it will be perceived that EI may be described in the opposite direction, by starting off the tangent EF, instead of ER, and also if AI were given, and E required, IE might be described either from IK or IN the parallel to AG.

The degree of curvature for EI is found thus: $a = 57.30\ m$ and $n =$ curvatures of AE and AI; then $\frac{a}{m} =$ Radius of AE and $\frac{a}{n}$ Radius of AI, and their difference $= \frac{am-an}{mn}$ and a divided by this result $= \frac{mn}{m-n}$; i. e. *multiply the curvatures together, and divide by their difference.*

NOTE. When the product of the two curvatures is very great and their difference small, a differential curvature will result which is too large to be used with the chord of 100 feet without occasioning a sensible error in the result, the proper method then is, to take smaller chords of 10 or 20 feet; reducing the curvature in a like proportion. Thus, if m be 5 and n be 6 $\frac{mn}{m-n}$ will be 30°; then instead of a cord of 100 feet, take 20 feet and a central angle of 6°.

Fig. 11.



PROPOSITION VIII.

Having a Curve HAE, (*Fig.* 11,) *and a line* IK *at a distance from it, to draw another Curve of a different radius, which shall touch the first Curve, and also the line* IK.

CONSTRUCTION.

Let L be the centre of HAE. Draw the tangent EF parallel to IK and through L and E draw LS, and make it equal to the radius of the other circle: from S with the radius SE describe the differential circle EI, cutting IK in I: join IS and draw LA parallel to it; complete the parallelogram ISLM; then will M be the centre of the circle required. For ML is equal to SI or SE, and LE to LA, and hence, MA is equal to LS or MI, and a circle described from M with radius MA will touch HAE in A, and because MI and LS are parallel, and EF and IK are parallel, the angle MIK is equal to LEF, which is a right angle, and therefore the circle AI touches IK in I.

APPLICATION.

It is evident from the mathematical solution, that the curve HAE must be continued until its tangent at the point E is parallel to IK, then from EF as a tangent the differential curve EI must be traced as directed in the preceding proposition until it intersects IK in I, the central angle ESI must then be ascertained from

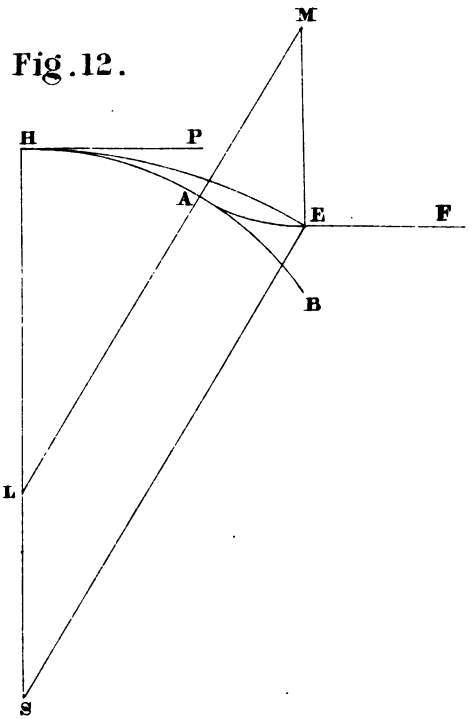
the index, (after completing the tangent as in Proposition V,) and the curve A E made equivalent to it; A will then be the P. C. C. sought, from which if a curve be described with the proper curvature, it will touch IK in the point I.

It is also evident, that if AI were given, and AE required that the centre S would fall upon N, and that IE must be described from the tangent IK in the opposite direction till it intersects EF.

It often happens, that the continuation HAE to the tangent EF is rendered difficult or impracticable by the roughness of the ground; recourse must then be had to Proposition X.

NOTE. The observations in the note attached to the preceding Proposition will apply to this, as well as several other succeeding Propositions.

Fig. 12.



PROPOSITION IX.

When the position of the tangent EF, Fig. 12, *is such that if produced it would cut* HAB, *it is evident that a curve which shall touch them both externally, must be turned in the opposite direction from* HAB. *This is termed a Reversed Curve, and may be described as follows.*

TAKE a point H in HAB, where the tangent HP is parallel to EF, through H and the centre L draw HLS and make it equal to the sum of the two radii HL and ME. From S, with radius HS, describe the integral curve HE intersecting EF in E, and complete the parallelogram LSEM. Then since ML equals ES or HS, and LA equals HL, MA must equal ME or LS, and since HP is parallel to EF, and ME to HL, the angle MEF equals LHP, which is a right angle. Therefore a circle described from the centre M with radius MA will touch EF in E.

The field operation is evident from the above construction, being strictly analogous to the preceding Proposition. The integral curvature is found as follows: $a = 57.30$, $m =$ curvature-AE, $n =$ curvature HA, then radius AE, $= \frac{a}{m}$, and radius HA $\frac{a}{n}$ and their sum $= \frac{ma+na}{mn}$, and a divided by this result gives $\frac{mn}{m+n}$; i. e. *multiply the curvatures together and divide by their sum.*

When H is inaccessible, as it often will be, resort must be had to the third case of the following Proposition.

Fig. 13.

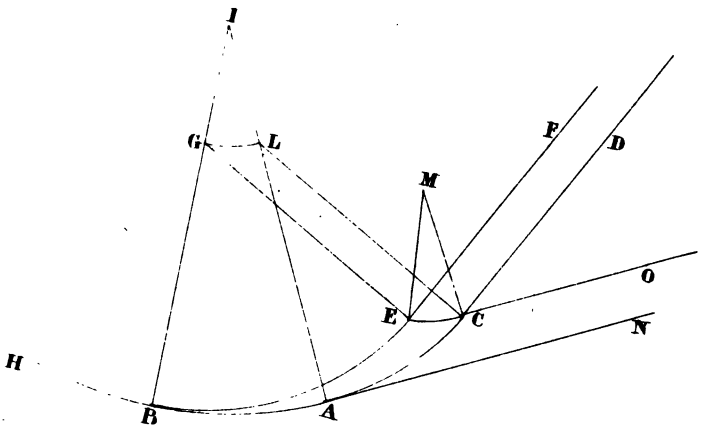
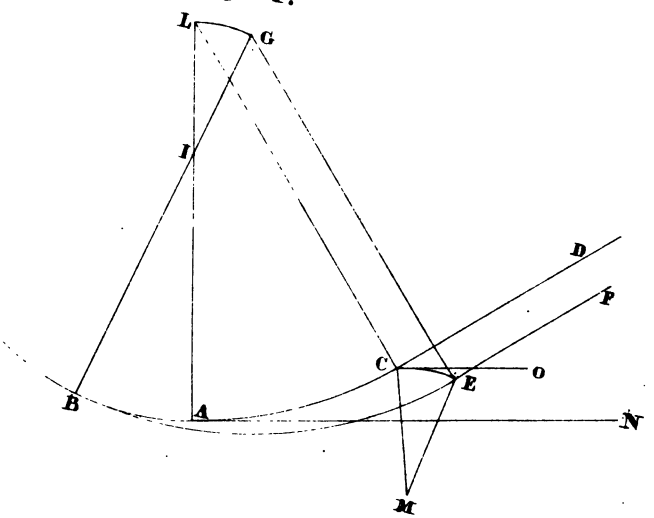


Fig. 14.



PROPOSITION X.

Having located a compound Curve HA, AC, *Fig.* 13, *terminating in a Tangent* CD, *it is required to change the point of compound curvature from* A *to* B *so that the Curve will terminate in a Tangent* EF *parallel to* CD.

LET I be the centre of HBA, and L the centre of AC, and draw ILA, then IL is the difference of the radii of the two curves: make CM equal and parallel to IL, and describe the differential curve CE, cutting EF in E. Draw IB parallel to ME, then will B be the new P. C. C. sought. For if BG be made equal to LA or LC, IG will be equal and parallel to ME, and consequently GL will be equal and parallel to CE, and GE will be equal and parallel to LC, and at right angles to EF, and hence, a circle described from G with the radius GB will make tangent upon EF at the point E.

APPLICATION.

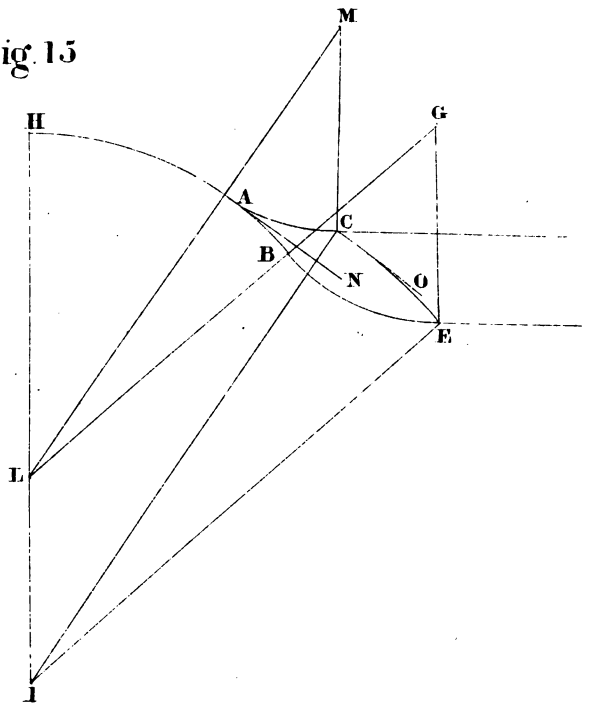
If, in starting from the point A, the index of the Transit be set at Zero, it will show upon completing the tangent at C the whole angle between AN and CD, and by turning it back again to Zero, the Telescope will assume a position CO parallel to AN: then from CO as a tangent describe the differential curve CE, until it intersects EF, and ascertain as in the last Proposition, the central angle of EC which is equivalent to AB,

and consequently the point B may be readily obtained by setting the transit over A, and retracing a portion of the curve equivalent to EC.

A second case of this Proposition occurs when the radius of AC, Fig. 14, is greater than that of HA, the effect of which is to throw the point M on the opposite side of EC, and consequently the curve must be turned in the opposite direction in passing from C to E.

A third form of this Proposition is presented in the reversed curve HAC, Fig. 15, which will be investigated on the next page.

Fig. 15



CASE THIRD.

When a reversed curve is presented instead of a compound one.—

LET the reversed curve HAC, Fig. 15 be described having its tangent CD, parallel to EF, and let it be required to transfer the P. R. C. from A and B so that BE shall make tangent upon EF.

From C and the tangent CO parallel to AN describe the integral curve CE intersecting EF in E and extend HA towards B until AB becomes equivalent to CE. B, will be the P. R. C. required.

The demonstration of this method is so strictly analogous to case 1st, that it is unnecessary to go into it, being derived from the same system of parallel lines as the other.

Fig. 16.

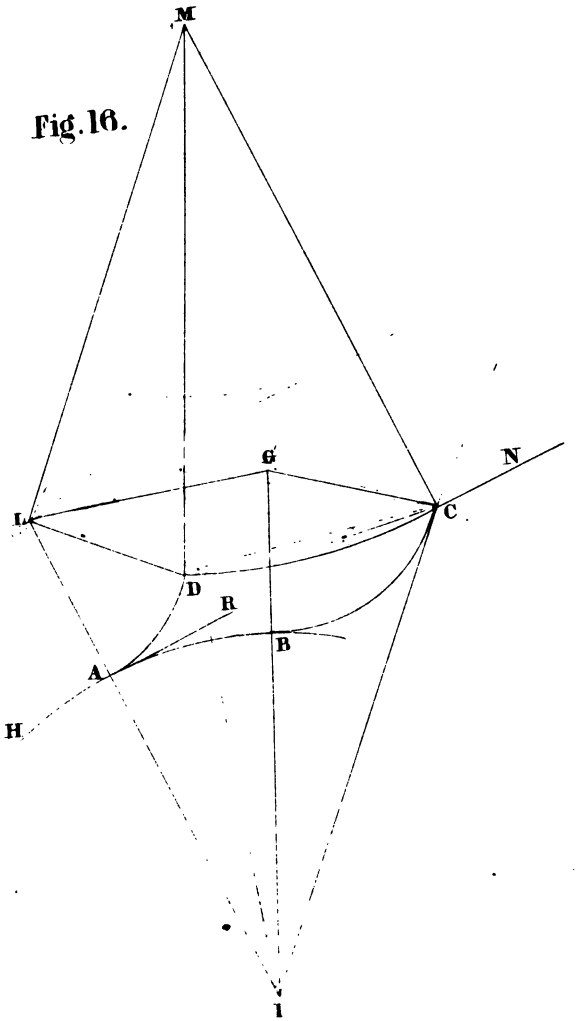


Fig. 17.

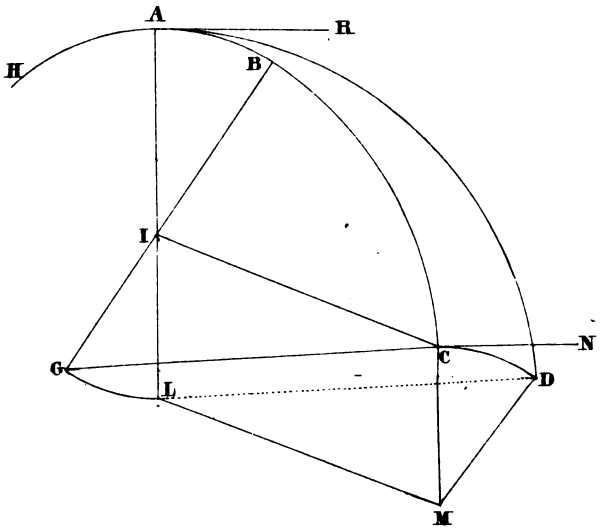
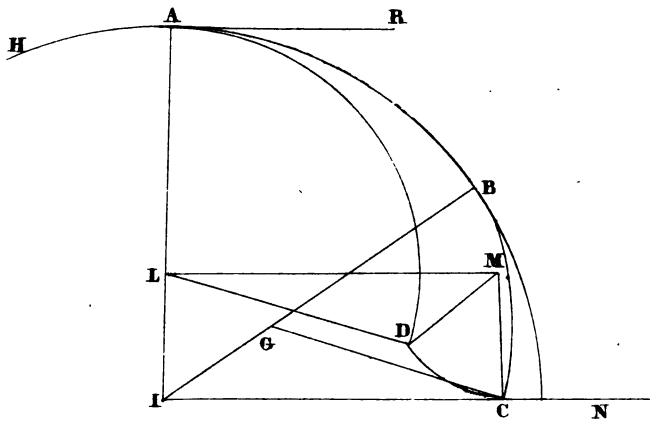


Fig. 18.



PROPOSITION XI.

Having located a Curve HAB, *Fig.* 16, 17 *and* 18, *to draw through a point* C, *another Curve which shall touch* HAB.

In HAB take any point A, and through A and the centre I draw AL, and make it equal to the radius of the required circle, and from L as centre, describe AD; join IC and complete the parallelogram ILMC, then will M be the centre of an integral circle, Fig. 16, and a differential circle, Fig. 17 and 18. From the centre M describe CD cutting AD in D, join MD and draw IB parallel to it, make BG equal to LA then will G be the centre of the curve BC.

For, since IL and IG are parallel and equal to MC and MD, GL must be parallel and equal to CD, and hence, GC must be parallel and equal to LD, i. e. to LA or GB; hence, a circle described from G with GB will pass through C as required.

APPLICATION.

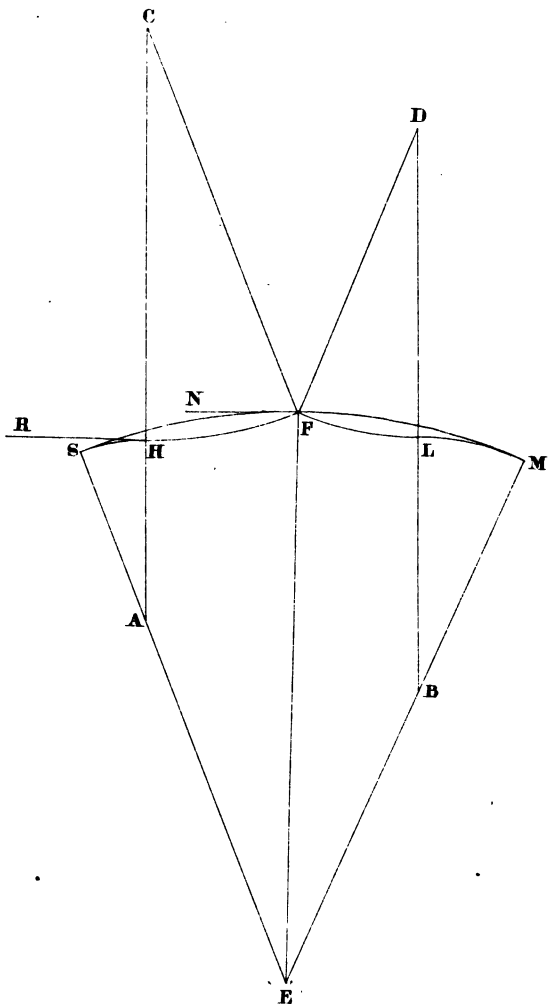
Since CM is parallel to AI, the tangent CN must be parallel to AR. Therefore, from any point A describe AD with the Transit, and from any convenient point in AD deflect to C, and by Proposition IV, page 13, bring the Telescope into the position CN, parallel to AR, then describe the differential or integral curve CD till it intersects AD; complete the tangent at D, and ascertain the central angle of CD and make

AB equivalent: from a tangent at B describe BC, which must pass through the point C.

Observe that in Fig. 17, CD is turned towards the same hand as AB, and in Fig. 16 and 18 towards the opposite hand. The reason is sufficiently evident from the position of the parallelogram ILMC.

Note.—The distinction of the several cases must be carefully observed. In Fig. 16, ABC is a *reversed* curve. In Fig. 17, 18, ABC is a compound curve. In Fig 17, C is outside of AB and the radius of BC is of course the greatest. In Fig. 18, C is inside of AB, and of course the radius of BC must be least

Fig. 19.



PROPOSITION XII.

Between two Curves already described, to draw a third Curve with a given Radius which shall touch them both.

CASE FIRST.

When all the Curves must be turned in the same direction.

LET HS and LM, Fig. 19, be the given curves of which A and B are the centres, it is required to describe a third curve SFM, with a given radius which shall touch the first two.

Take any point H in HS, and a corresponding point L in LM; draw AH and BL, and produce them till AC and BD are each equal to the radius of the third curve. From C and D as centres describes the differential curves HF and LF, and from their intersection draw FE parallel to AC and BD, and complete the parallelogram ACFE or BDFE, then will E be the centre of a curve SFM which shall touch the other curves in S and M.

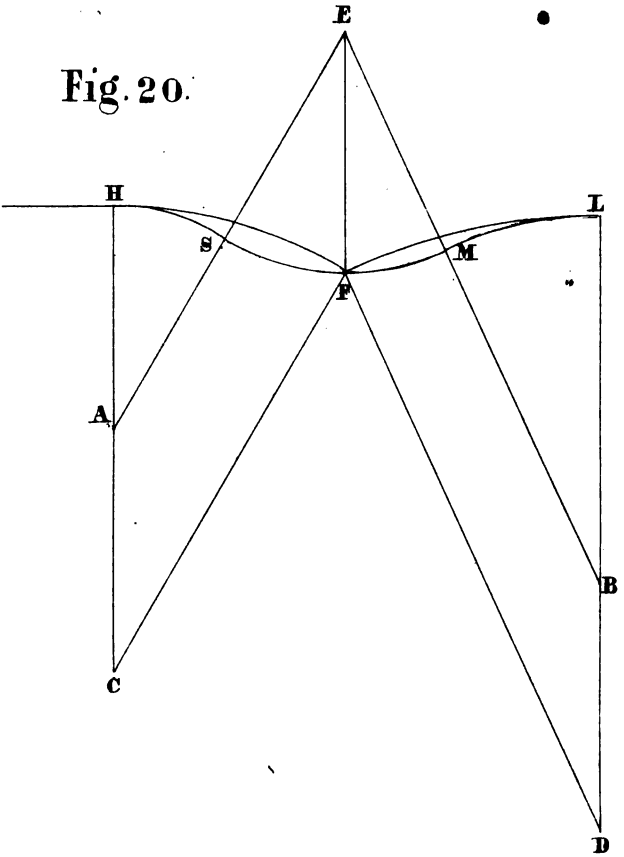
For AE being equal to CF or CH, and AS to AH, ES must be equal to AC, which by construction is equal to EF. Therefore EF is equal to ES, and by the

same reasoning EM may be proved to be equal to EF; therefore E is the centre of the curve SFM, and since SE and EM pass through the centres A and B, SFM must touch HS and LM in S and M respectively.

APPLICATION.

Since FE is parallel to HA, the tangent FN must be parallel to HR. Therefore from any two corresponding points, H and L, describe the differential curves HF and LF and from their intersection and with a tangent parallel to HR describe the required curve each way till it touches the others as required.

Fig. 20.



CASE SECOND.

When HS *and* ML, *Fig.* 20, *are turned in the same direction and it is required that* SM *shall be turned in the contrary direction.*

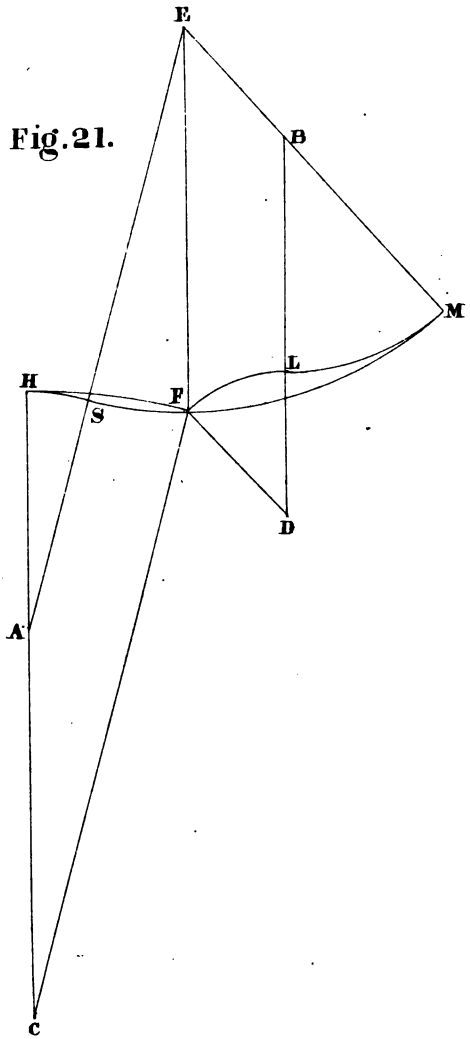
JOIN HA and LB as before, and produce them till AC and BD are equal to the radius of the third circle.

Describe the integral circles HF, LF; draw FE and complete the parallelograms as before, and E will be the centre of the circle required.

The demonstration of this case is so analogous to the last, that it need not be inserted here.

The field operation is also analogous to the last. The integral curves HF and LF must be turned in the same direction as HS or LM.

Fig. 21.



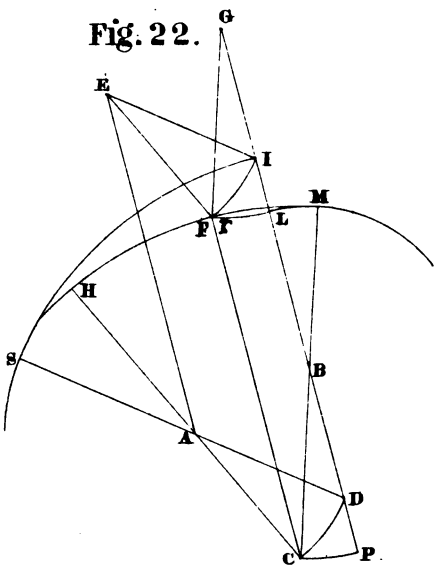
CASE THIRD.

Where HS *and* ML, *Fig.* 21, *are turned in opposite directions.*

TAKE H and L as before, and draw CA and BD in such a manner that CH shall be the radius of an integral and DL of a differential curve, to HS and LM respectively, complete the parallelograms as before, and E will be the centre of the circle required, *which must be turned in the same direction as the curve to which the differential radius was applied.*

NOTE. It will be observed, generally, of these three cases, that integral curves are turned in the same direction as the primary curves, and differential curves in the opposite direction, and that the curve SM is always turned in the opposite direction to the auxiliary curves used in constructing it.

Fig. 22.

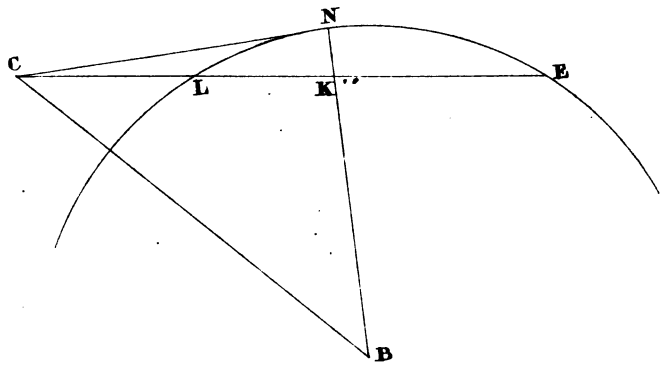


PROPOSITION XIII.

However easy of solution the three last cases may appear to the student in his closet, yet so rude is the drawing board with which nature furnishes the tired practitioner in the field, that it is not always practicable to assume the points H *and* L, *especially in case* 1*st, where both of them lie without the line of location. In this dilemma the principle of trial and correction may be successfully applied as follows.*

CHOOSE any point S, Fig. 22 which by trial may be not far from the true point of contact for the middle curve and describe SI with the requisite curvature. From I and a tangent parallel to the tangent at S, describe the differential curve IF equivalent to CD. Take L in LM corresponding to I in SI and from a tangent at L describe the differential LF equivalent to CP. the intersection of LF and IF will be a point in the required curve HFM, the arc IF will be equivalent to SH and LF will be equivalent to LM. The tangent to HFM at F will be parallel to the Tangent at I to SI or at L to LM. The same mode of solution may be applied to case 2nd by using integral curves and to case 3rd by using a differential and integral together. These cases will seldom however occur in practice.

Fig. 23.



PROPOSITION XIV.

To draw a Tangent to a Curve from a point without it.

Let LNE Fig. 23, and C the point without it. Draw any line CLE cutting the curve in L and E and make $CK^2 = CL\ CE$. From C as centre with radius CK describe KN cutting LNE in N. N will be the point of Tangent required as shown by Prop. 8, page 8.

APPLICATION.

As the location advances in the direction EN take any point E which may be the nearest full station to the required P. T. and from the tangent at E deflect to C.

Measure EC and from the deflection above obtained calculate the value of EL which subtract from EC the remainder will be CL.

Multiply CL by CE and the square root of the product will be CK and CE — CK = EK which must be measured from E and fixed by the transit.

Divide 57.30 by CK, the result will be the curvature of KN. From K with a tangent at right angles to CK locate KN and find its intersection with EL.

If EL be less than 100 feet, or if it be small in proportion to CL, the curve KN will not differ sensibly from the ordinate at that point as found by Prop. XVIII. Fig. 28, which may therefore be substituted for it in a majority of cases or EN may be assumed

to be equal to EK in very many cases of which the limit will soon be ascertained in practice.

Example. Suppose LNE be a 4° curve and the deflection of EC from a tangent at E be 1°.88 and the distance from E to C 212 feet.

Then $EL = \frac{188}{2} = 94$ and $CL = 212 - 94 = 118$

$CK = \sqrt{212 \times 118} = 158.16$ and $EK = 53.84$.

The ordinate of EL at the point K will be 0.75. The curvature of KN 36.3° and its departure from the ordinate only .002 in a direction nearly parallel to the curve and therefore not affecting its accuracy

Fig. 24.

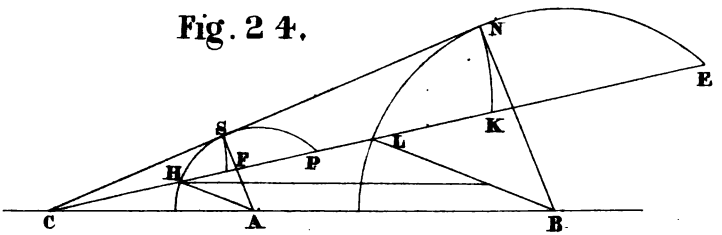
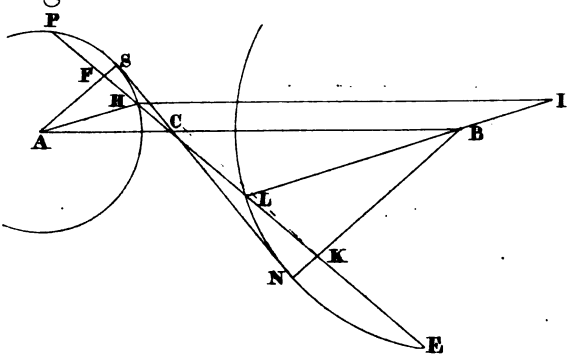


Fig. 25



PROPOSITION XV.

To draw a Tangent to two Curves already located.

TAKE any point H, (Fig. 24 and 25,) in the circle HS, and take L a corresponding point in the other circle LN, and measure HL. Then suppose HI drawn parallel to AB and take CH: HL: HA: LI,

i. e. in Fig. 24, $CH = HL \times \frac{n}{m-n}$

And in Fig. 25, $CH = HL \times \frac{n}{m+n}$

Then C will be a point in the common Tangent which may be drawn as directed in Proposition XIV.

For HAC and LIH are similar triangles, and LI: HA: : HI: CA, but LI, HA and HI are constant and, therefore, C is constant, and any line drawn through C to meet both circles will make the same angle with the radii at the point of meeting, and therefore, if it touch one circle, it will touch both.

It will not be necessary to locate the point C, but having found it, measure HL and the angle which it makes with the tangent at H or L, the other elements may be found and the position of K and F determined by calculation.

If the two curves are of equal curvature, the length of CH Fig. 24 becomes infinite, and IS and KN will coincide with the middle ordinates of HP and LE.

Example. Suppose HL be 326 feet, its angle with the tangent at H or L 2.14°, the curvature of HP 4° and of LE 3°.

Then Fig. 24. $HC = 326 \times \frac{3}{4-3} = 978$ feet,

and $LC = 1304$,

$HP = \frac{2.14}{2} = 107$ and CP 1085,

$CF = \sqrt{978 \times 1085} = 1030.0$ and $HF = 52$,

$LE = \frac{2.14}{1.5} = 143$ and CE ± 1447.

$CK = \sqrt{1304 \times 1447} = 1373.64$ and $LK = 69.64$,

And in Fig. 25 $HC = 326 \times \frac{3}{4+3}$ 139.7 and LC=186.3,

CP=246.7. $CF = \sqrt{246.7 \times 139.7} = 185.6$ & HF=45.9,

CE=329.3 $CK \sqrt{329.3 \times 186.3} = 247.7$ & LK=61.4.

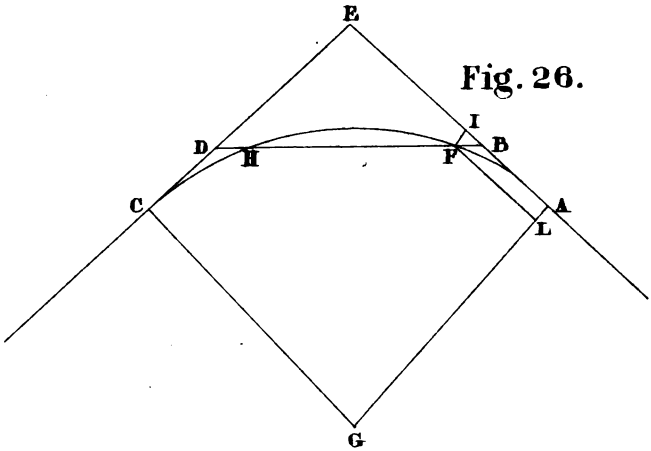


Fig. 26.

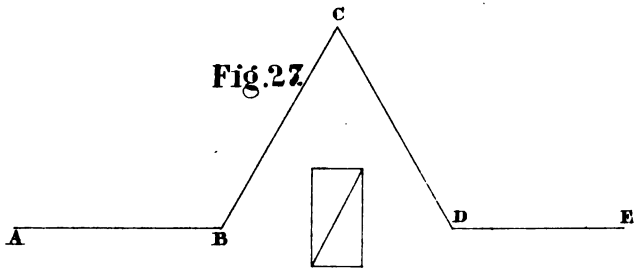


Fig. 27.

PROPOSITION XVI.

Between two Tangents to trace a curve which shall pass through a given point.

SUPPOSE AB and CD Fig. 26 to be two tangents permanently fixed, as for instance the centre lines of two streets or other highways which it is convenient to occupy and let F be the given point which may be at a minimum distance from the corner of some building which it is inexpedient to remove.

Suppose AB and CD produced to meet in E and the line EG drawn bisecting the angle at E. Through F draw BFD, at right angles to EG. By measuring BF and FD we have BH which is equal to FD and by prop. 8 we have BF. BH=BA^2 or DC^2.

In solving this case instrumentally in the field it will not be necessary to produce the tangents till they meet but the angle they contain can be measured by prop. III. and BF and FD can be run and measured by prop. IV. With BF and FD find BA as above. Measure FA and the angle FAB which divided by FA will give the curvature of AFHC.

If AF does not much exceed 100 and BFA is not greater than 10° the curvature thus found will be sufficiently accurate. But if greater precision is required, measure FL and LA or their equals FI and IA and then $\frac{FL^2}{LA}+LA=2AG$ the diameter of the circle and 57.30 ÷ AG = curvature required which if greater than 10° must be applied as directed in the Note, Page, 20.

PROPOSITION XVII.

Of passing around obstructions by an equalateral Triangle.

When this operation is nicely performed it is better than offsetting which requires more time and is less accurate.

Suppose the line AB Fig. 27 to be run from A to B and on reversing the telescope an impenetrable obstacle presents itself in a building without an opening in the direction of the sight. If AB be a straight line the index should be set at Zero when the back sight is taken upon A: Deflect 60° to the left in the direction BC, measure BC and fix the point C. Remove the instrument to C and set in such a manner as that a back sight may be taken to B without reversing the telescope. Deflect 60° more to the left, the index will now stand at 60° on the second quadrant. Measure CD and set the instrument at D in the same manner as at C and take a back sight without reversing: Deflect 60° more to left and the index will have come round to Zero, 180° from whence it started at B and the direction of the telescope will be that of a back sight upon B.

But suppose AB were a curve turning to the right the index would then stand at some point say 16.°50 on 1st right hand quadrant which we will suppose to be its true place when arranged for a forward sight upon D. Turning 60 to left would give us 43.°10 on the 1st left hand quadrant when the telescope pointed to C. Deflecting 60° at C would bring the index to 76.50 on the 2nd quadrant and 60° more at D would bring us round to 16.°50 with a back sight upon B precisely as if the curve had been carried directly through from B to D.

Fig. 28.

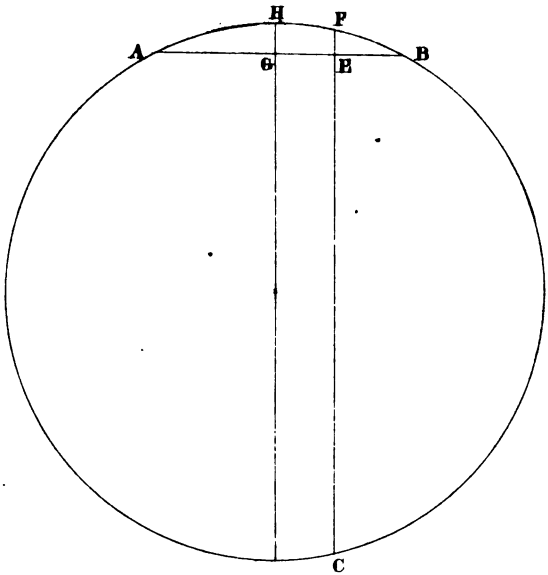
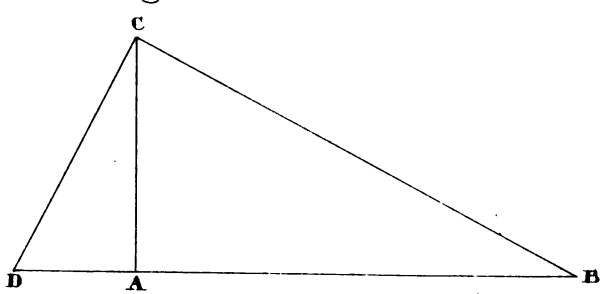


Fig. 29.



PROPOSITION XVIII.

To find the Ordinate EF, *Fig.* 28, *at any point in a given chord* AB, *the diameter of the circle being known.*

PUT AE = a, EB = b, GH = v, EF = x and D = diameter. Then, EC = D—$2v$—x and ab = Dx —$2vx$—x^2.

Now $2vx$—x^2 is too small to affect the result in any case of location; it may therefore be omitted and the equation becomes ab = Dx. But D = $\frac{114.6}{m}$; therefore ab = $\frac{114.6x}{m}$; and x = ab × $\frac{m}{114.6}$ = abm × $\frac{1}{114.6}$.

Now if ab be given in parts of a chain of 66 feet and the value of x be required in inches, then x = abm + $\frac{792}{114.6}$ = abm × 6.9.

But if the chain be 100 feet and x be required as before, then x = abm × $\frac{1200}{114.6}$ = abm × 10.5.

And if the chain be 100 feet, and x be required in feet, then x = abm × $\frac{100}{114.6}$ = abm × .875.

NOTE. It must be remembered (see Explanations, No. 2,) that the chord of 100 parts is considered as a *unit*, and therefore, the decimal point in a and b must be so placed as to diminish their value in like proportion.

PROPOSITION XIX.

To measure the Width of a River or distance to any inaccessible object in the line of survey.

LET AB, Fig. 29, be part of a line of survey in which A and B are on opposite sides of a river.

From A at right angles to AB lay off any convenient distance AC, so that B may be seen from C; remove the instrument to C, and lay off CD at right angles to CB, fixing the point D in AB produced, and measure DA.

Then AB is a third proportional to DA and AC and of course $\frac{AC^2}{DA} = AB$.

On page 48 the reader will find the table of chords referred to in the preface. The numbers in the table are the ratios of the base to the side of an isosceles triangle for every degree of verticle angle. It would be difficult to give a specific account of its various uses, since it may be applied to every case which can be resolved into an isosceles triangle of which one side and angle are known.

PROPOSITION XX.

To find approximately the departure of a curve from its tangent, or of one curve from another.

THE departure of a curve from its tangent at right angles to the tangent is equal to its versed sine, being in fact one end of a parallelogram of which the versed sine forms the other end, and the sine and a part of the tangent the sides. It is therefore mathematically expressed by $\frac{S^2}{D-V}$.

If for this formula which is inconvenient in practice we substituted $d = \frac{L^2}{D}$ we shall have an equation as nearly correct as that used for finding the ordinates Prop. XVIII., but as this calculation is required on a much larger scale, the error of result is increased in proportion, so that it can only be considered an approximate solution.

Now $D = \frac{114.6}{m}$ and hence $\frac{L^2}{D} = \frac{L^2 m}{114.6} = L^2 m \times \frac{1}{114.6}$.

But when the unit for measurement for L is a chain of 100 feet, and the result is required in feet, the equation becomes $L^2 m \times \frac{100}{114.6} = L^2 m \times 0.875 = \frac{7}{8}$ of $L^2 m$.

i. e. Square the number of stations in the curve, multiply by the curvature, and subtract one eighth of the product.

If instead of L and M being given to find d we have d and m to find L, the equation becomes $L = \sqrt{\frac{d}{875m}}$, and if L and d be given to find m we have $m = \frac{d}{.875L^2}$.

Again if we wish to ascertain the departure of one curve from another we have,

$$.875\ L^2 m - .875 L^2 m = L^2 \times (m - n) \times .875.$$

i. e. Square the number of stations in either curve, multiply by the difference of curvature, and take of one eighth of the product.

The result will be the departure of the two curves from each other, one line at right angles to their common tangent.

TABLE OF CHORDS. RADIUS BEING UNITY.

Deg.	0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170
0		.1743	.3473	.5176	.6840	.8452	1.000	1·1471	1.2856	1.4142	1.5321	1.6383	1.7320	1.8126	1.8794	1.9320	1.9696	1.9924
1	.0174	.1917	.3645	.5345	.7004	.8610	1.0151	1.1614	1.2989	1.4265	1.5433	1.6482	1.7407	1.8200	1·8853	1.9365	1.9726	1.9938
2	·0349	.2091	.3816	.5513	.7167	.8767	1.0301	1.1756	1.3121	1.4387	1.5542	1.6581	1.7492	1.8270	1·8910	1.9406	1.9754	1.9952
3	.0524	.2264	.3987	.5680	.7330	.8924	1.0450	1.1896	1.3252	1.4517	1.5652	1.6673	1.7576	1.8341	1.8967	1.9447	1·9780	1.9963
4	.0698	.2437	.4158	.5848	.7492	.9080	1.0598	1.2036	1.3384	1.4627	1.5760	1.6774	1.7659	1.8410	1.9020	1.9487	1.9805	1.9973
5	.0872	.2611	.4329	.6014	.7654	.9236	1.0746	1.2176	1.3512	1.4746	1.5867	1.6868	1.7740	1.8478	1.9074	1.9526	1.9830	1.9981
6	.1047	.2784	.4499	.6180	.7815	.9389	1.0892	1.2313	1.3640	1.4863	1.5973	1.6961	1.7820	1.8544	1.9126	1.9563	1.9851	1.9989
7	.1221	.2956	.4669	.6346	.7975	.9543	1.1039	1.2450	1.3767	1.4980	1.6077	1.7053	1.7897	1.8609	1.9177	1.9600	1.9871	1.9994
8	.1395	.3128	.4838	.6512	.8135	.9696	1.1184	1.2586	1.3893	1.5094	1.6180	1·7146	1.7976	1.8672	1.9226	1.9632	1.9890	1.9997
9	.1569	.3301	.5008	.6676	.8294	.9848	1.1328	1.2722	1.4018	1.5208	1.6282	1.7232	1.8052	1.8734	1.9273	1.9665	1.9910	2.0000

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